Lecture III: Multi-wavelength and multi-messenger astrophysics

Roland Crocker
CIPSS 2023

roland.crocker@anu.edu.au
☞ EMAIL ME ☜

Australian National University

Hadronic interactions

- We saw in the previous lecture that hadronic collisions lead to the production of charged and neutral mesons, dominantly pions:

$$
\begin{aligned}
p + p &\rightarrow p + p + \pi^0 \\
p + p &\rightarrow p + n + \pi^+ \\
p + p &\rightarrow p + p + \pi^+ + \pi^- \\
p + p &\rightarrow p + p + \pi^+ + \pi^- + \pi^0
\end{aligned}
$$

- Neutral mesons generally decay to gamma-rays; charged mesons generally decay to final state positrons, electrons and neutrinos

Hadronic interactions

- The charged pions decay via the weak force into (mostly), first, muons and accompanying mu neutrinos; the muons then subsequently decay to electrons/positrons and further neutrinos (so as to conserve overall lepton flavor).

$$
\begin{array}{lllll}
\pi^0 & \to & \gamma + \gamma & & \\
\pi^+ & \to & \mu^+ + \nu_\mu & \text{then} & \mu^+ \to \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \\
\pi^- & \to & \mu^- + \bar{\nu}_\mu & \text{then} & \mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu
\end{array}
$$

Charged mesons and astrophysical neutrinos

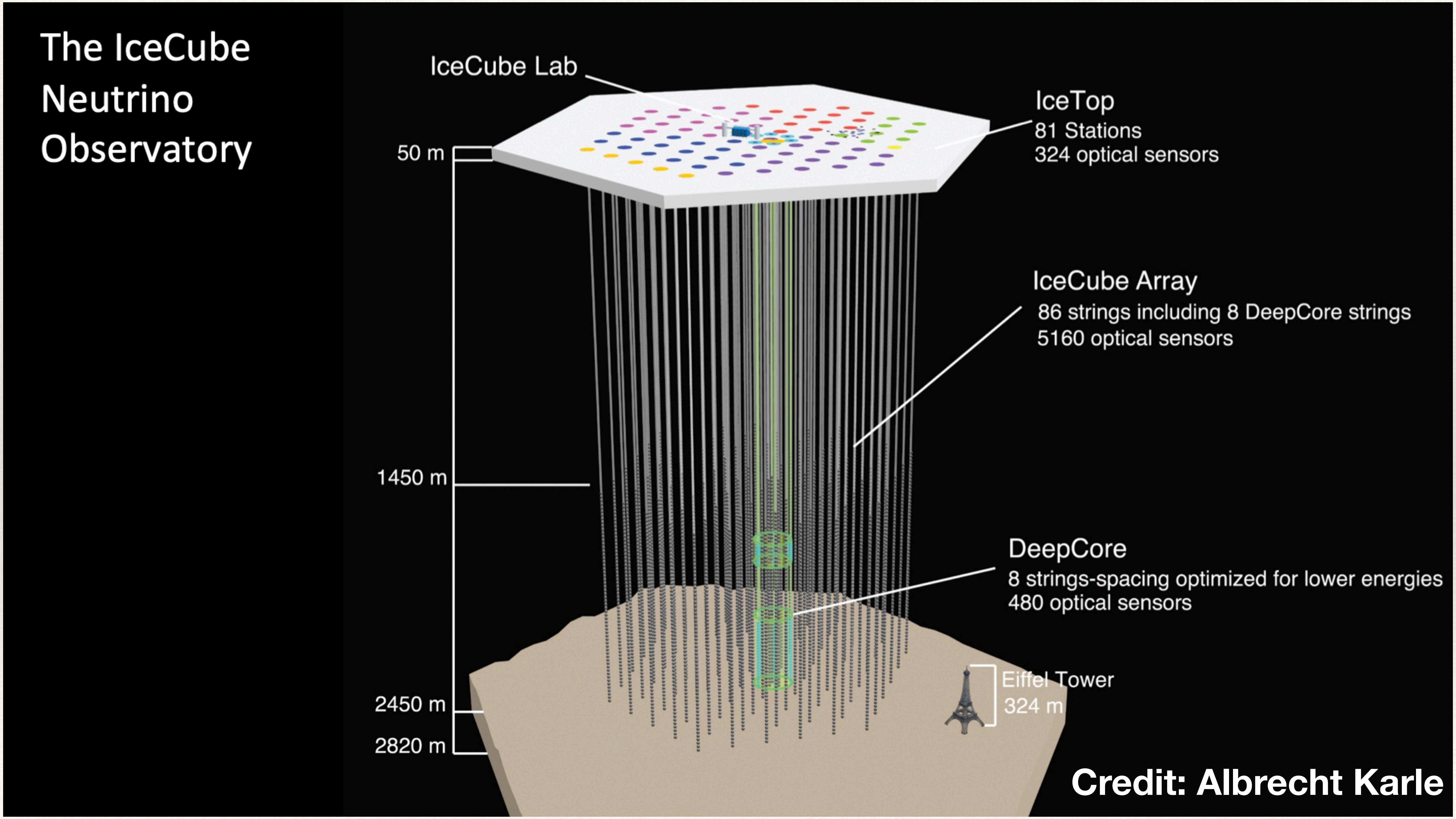
- Again disregarding the distinction between neutrinos and anti-neutrinos, the charged pion decays lead to neutrino production in the flavor ratio: $\nu_e : \nu_\mu : \nu_\tau = 1:2:0\,.$
- However, given that the lepton **flavor** is not exactly conserved, neutrinos have the strange ability to **oscillate** from one flavor to another. This process is observed for both ~MeV neutrinos created in nuclear reactions in the Sun and for ~GeV neutrinos created in the decay of pions produced by cosmic ray impacts in the atmosphere.
- After propagation, oscillations redistribute the neutrino flavors approximately as $\nu_e : \nu_\mu : \nu_\tau = 1:1:1\,.$

Astrophysical neutrinos

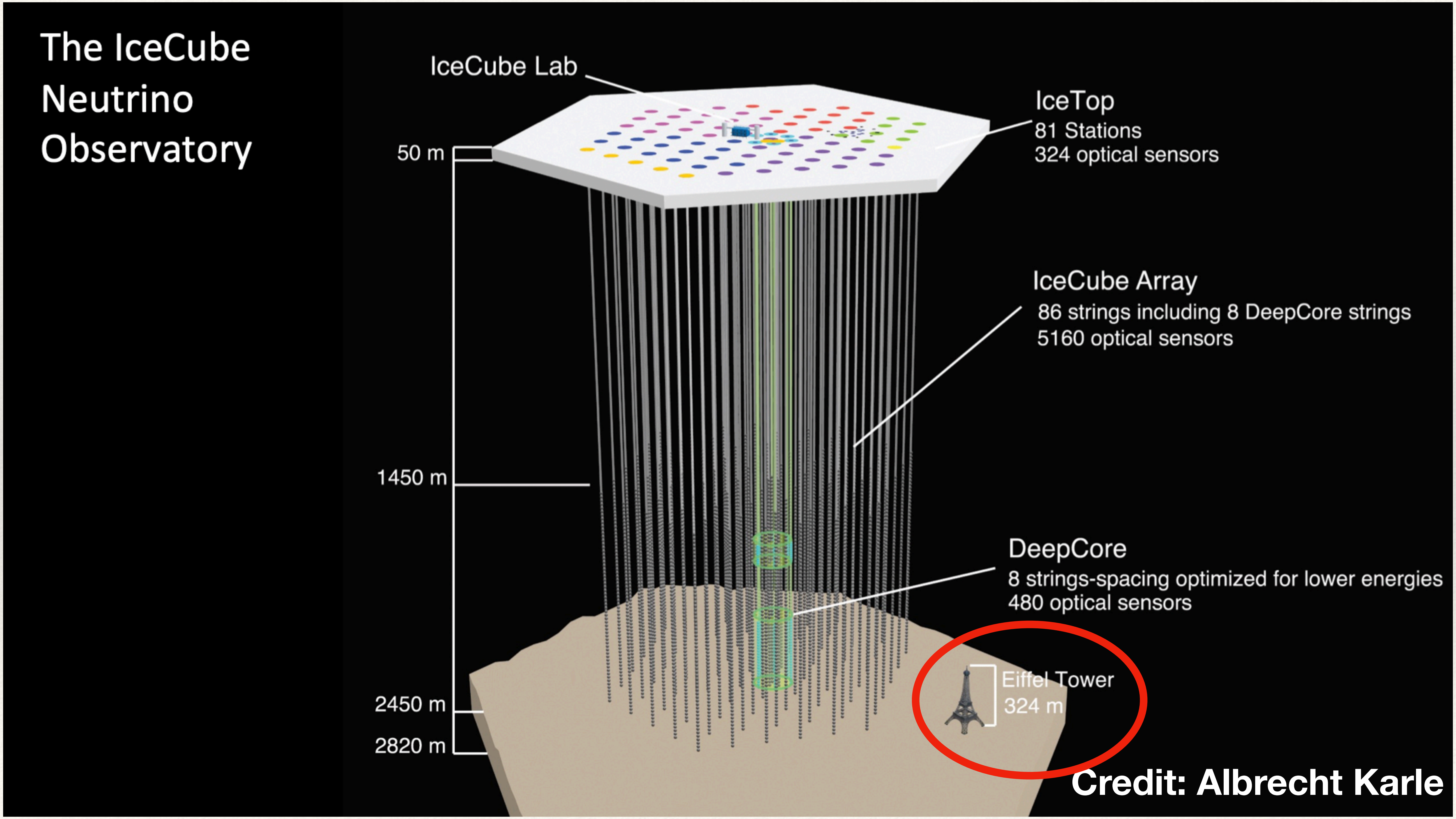
Astrophysical neutrinos

- Note that for many astrophysical objects observed to be γ-ray emitters – e.g., particular Galactic supernova remnants (SNRs) – it is unclear whether the high-energy photons are hadronic (i.e., of pp origin) or leptonic (i.e., from IC or bremsstrahlung emission) origin.
- However, if we could identify the concomitant neutrino signal from such objects, this would constitute a 'smoking gun' for the hadronic process.
- The problem with this neat idea is that neutrinos interact so weakly: one needs a huge detector volume, ~ 1 km^3 in order to have a decent hope of detecting non-solar astrophysical neutrinos.
- This technology has, in fact, been realised within the last decade by the IceCube detector located in the ice below the South Pole station; IceCube announced the discovery of astrophysical neutrinos (against the strong background of the 'atmospheric' neutrinos created by cosmic rays briefly discussed above) in 2013, an amazing achievement!

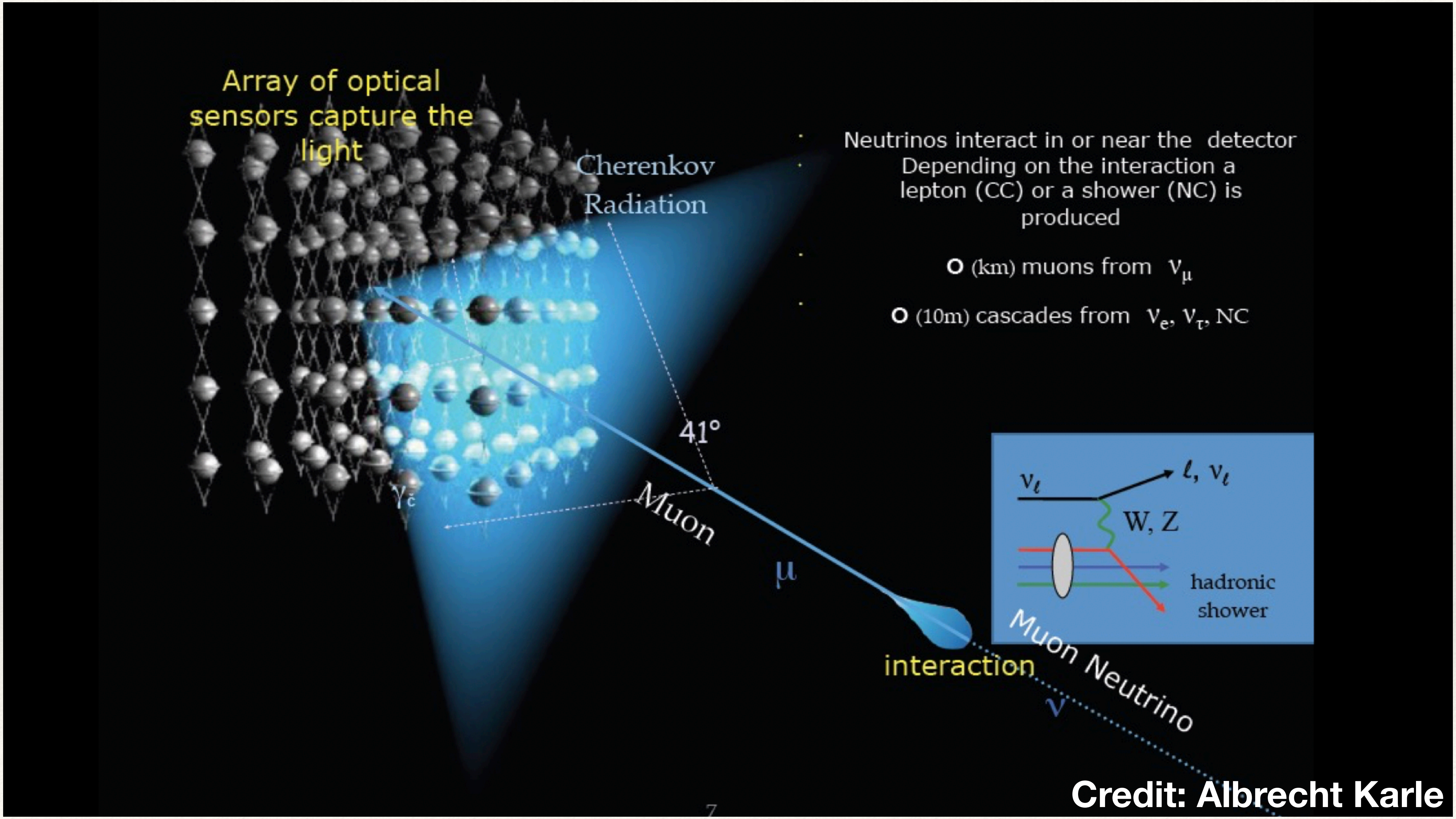
Astrophysical neutrinos



Astrophysical neutrinos

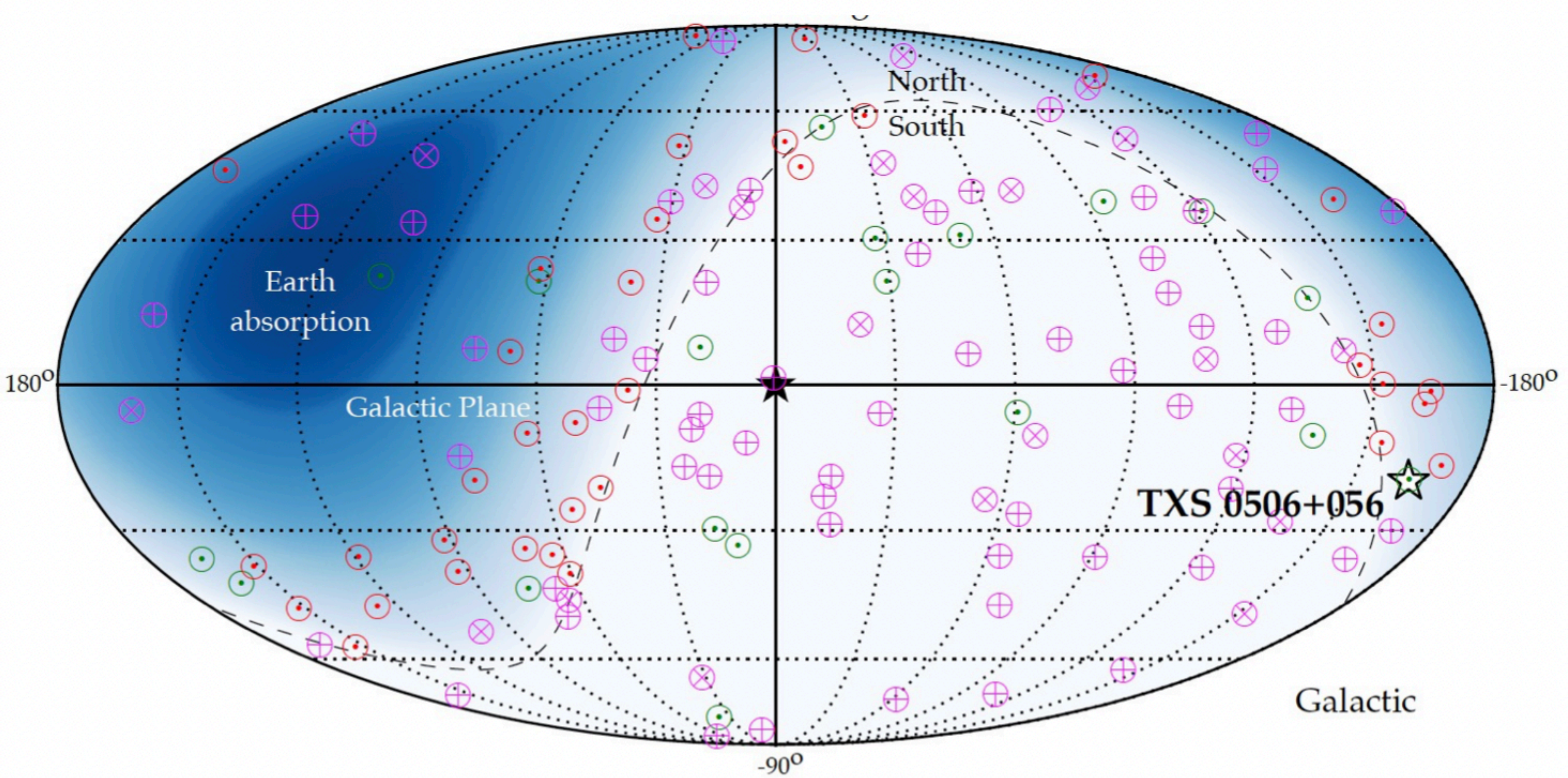


Astrophysical neutrinos



Astrophysical neutrinos

10 years of IceCube: The high energy neutrino sky



Credit: Albrecht Karle

Hadronic interactions

- Another important consequence of charged meson production is the existence of final state electrons and positrons

'Secondary electrons'

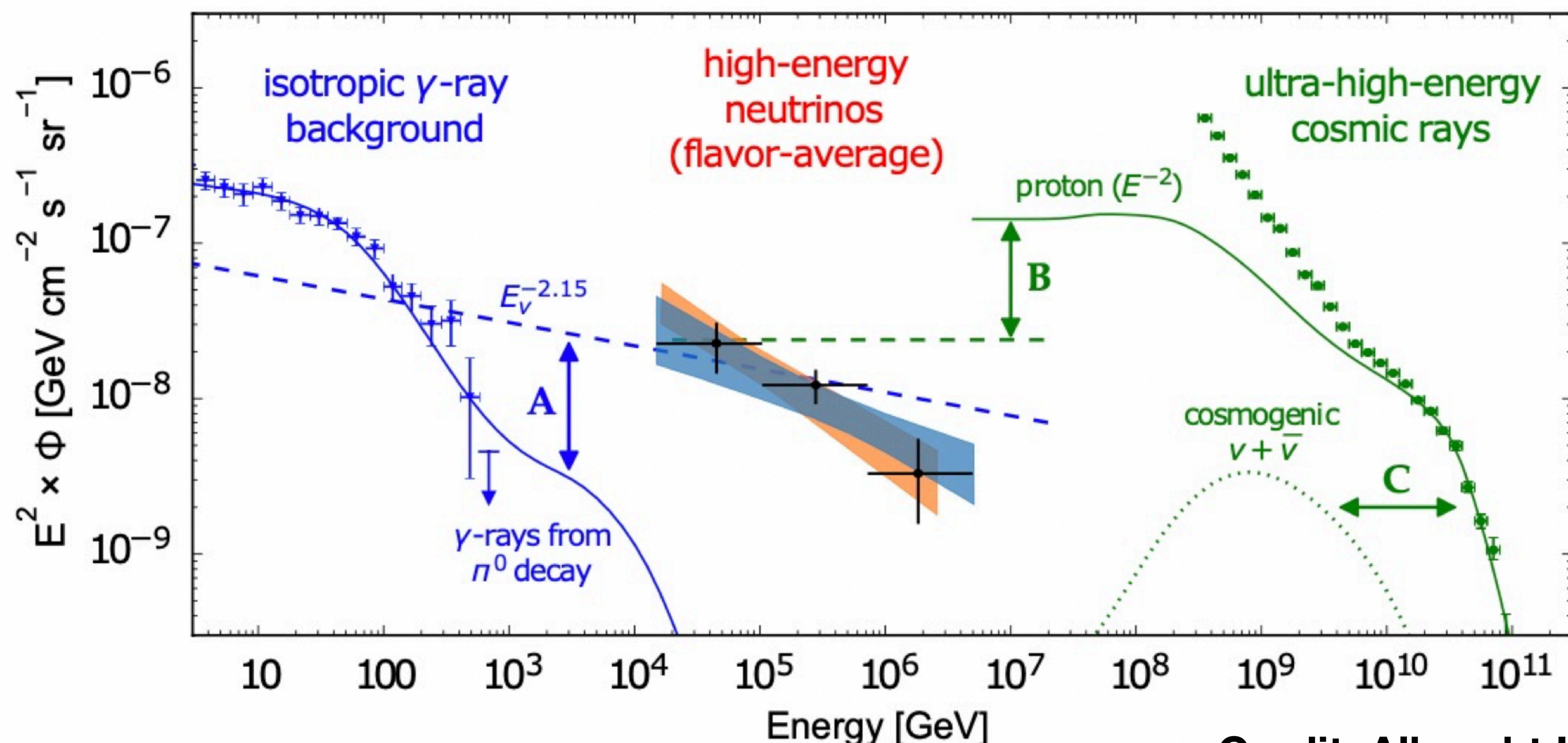
$$\begin{array}{lcl} \pi^0 & \rightarrow & \gamma + \gamma \\ \pi^+ & \rightarrow & \mu^+ + \nu_\mu \quad \text{then} \quad \mu^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \\ \pi^- & \rightarrow & \mu^- + \bar{\nu}_\mu \quad \text{then} \quad \mu^- \rightarrow \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu \end{array}$$

Hadronic interactions

- Another important consequence of charged meson production in pp collisions is the existence of final state electrons and positrons
- Astronomers call these ‘secondary electrons’
- In some astrophysical environments, fast cooling (via synchrotron or inverse Compton radiation) can prevent ‘primary’ electrons from being accelerated to high energies, while primary protons can be accelerated because their losses are slower
- These primary protons then go on the generate secondary electrons via their pp collisions
- Of course, such secondaries will be accompanied by neutrinos

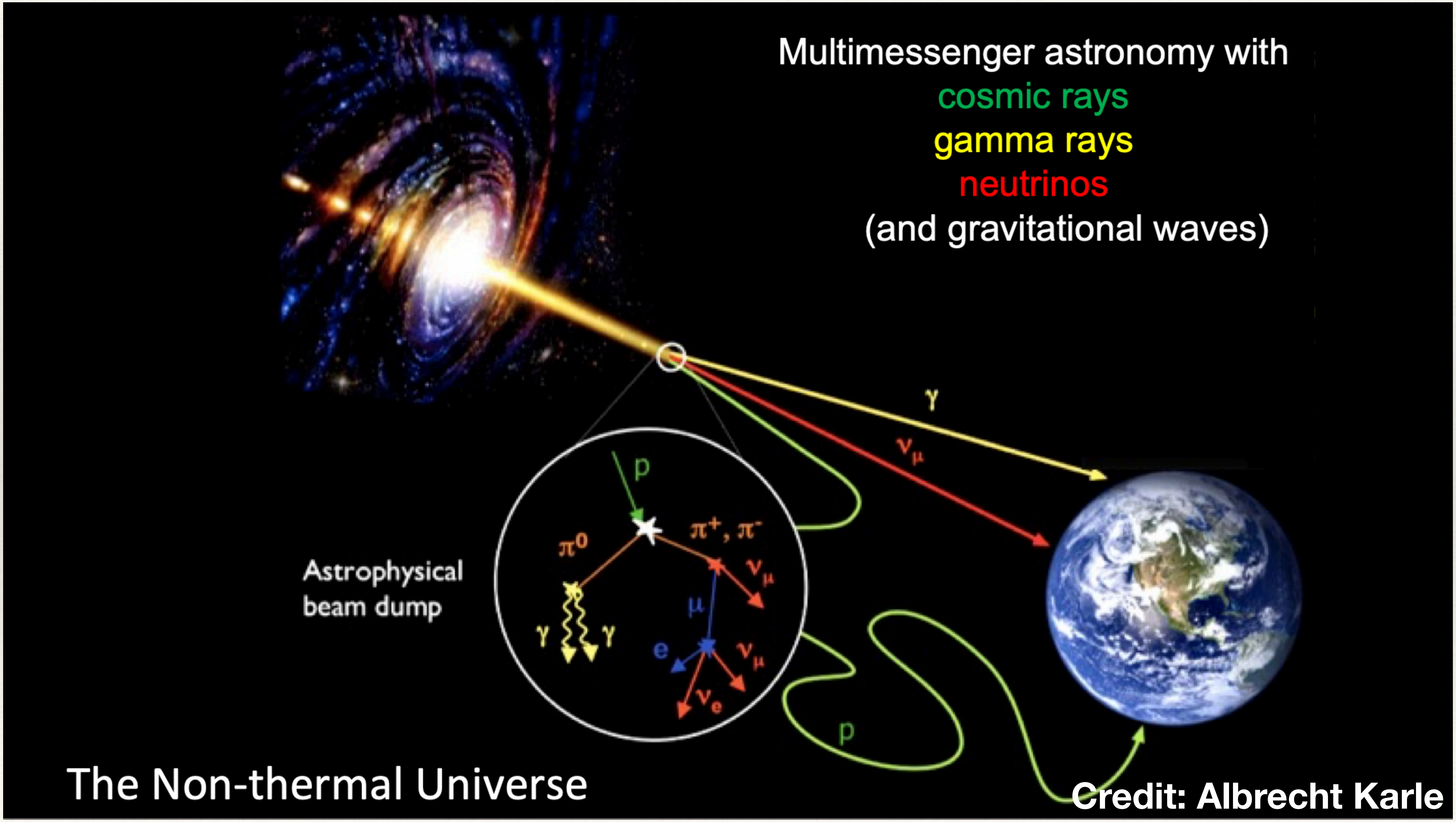
Putting it all together…

‘Multi-messenger’ astronomy



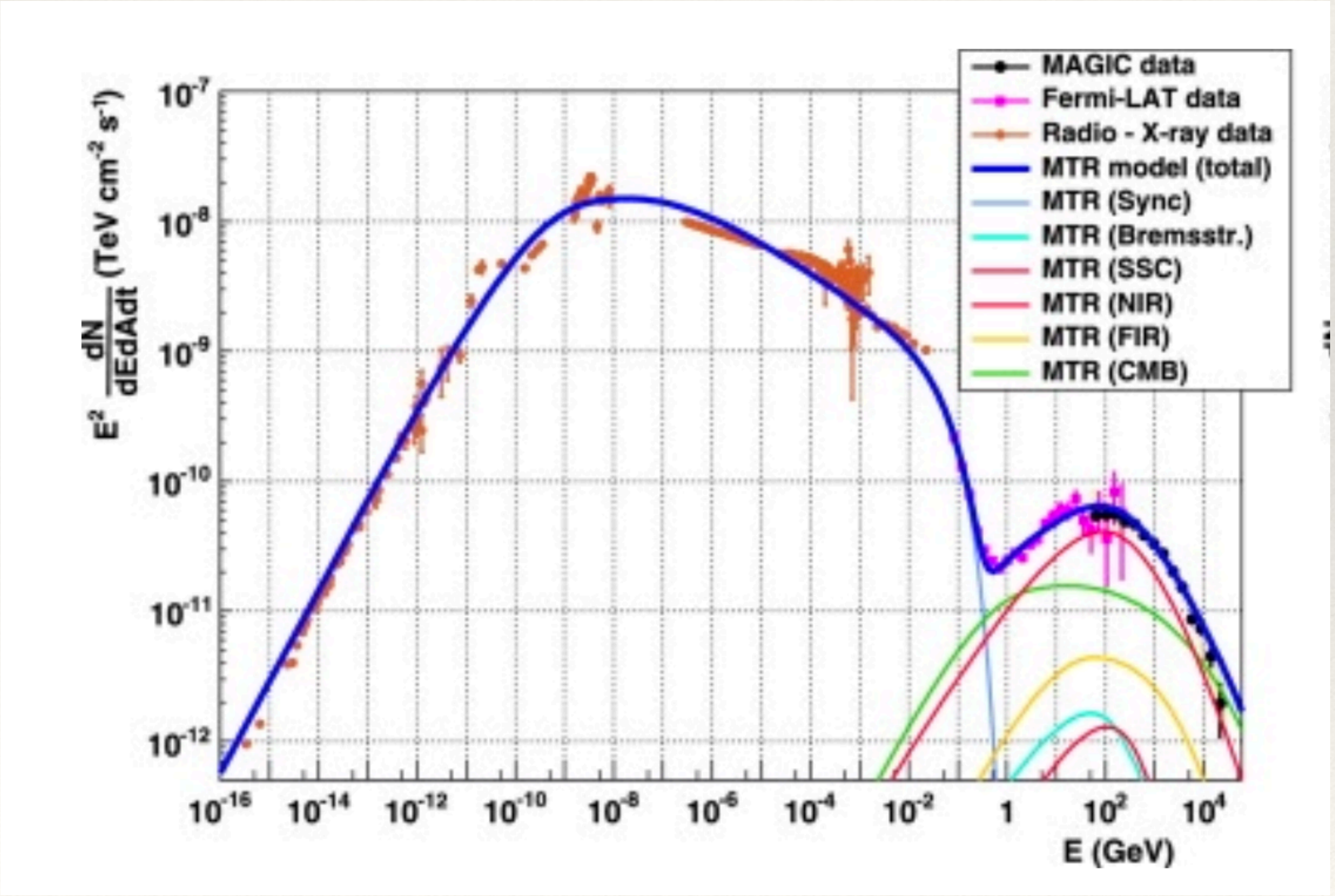
Credit: Albrecht Karle

‘Multi-messenger’ astronomy



General problem

How can we make sense of the broadband, non-thermal spectra of astrophysical sources and regions?



Broadband spectrum of the Crab nebula; model and data (reproduced from Aleksic et al. 2015).

Note the dynamic range of the measurements

General problem

We have already recently seen how γ-ray observations of the Galactic plane indicate the Milky Way is suffused by a population of cosmic ray protons and heaver ions with a similar spectrum and normalisation to that detected directly at the Earth.

However, such a situation of being able to perform such a cross-validation is unusual: for distant astrophysical objects, our only information about them is mediated by the radiation they emit; we have no 'direct' access to their particle populations.

This prompts us to consider the question: say we observe a certain (non-thermal) broadband spectrum of radiation from a particular source or astrophysical region; what can we learn from this? In particular, what can we learn about the object's in situ non-thermal particle population from the observed radiation?

In addition, what can we learn about the conditions in the object or region from this radiation?

General problem

We have already talked about a number of radiative processes – synchrotron emission, bremsstrahlung, inverse Compton radiation, hadronic γ-ray production – that affect non-thermal particle populations.

Such radiative processes are important for two connected reasons:

i) in general, they convey information about an object's particle population to us as discussed below

ii) because they are radiative, they carry away energy from an object's non-thermal particle population and, therefore, such processes themselves help to shape the broadband distribution of these particles.

General problem

- In general, the relative importance of different radiative processes changes with particle energy and is controlled by various environmental parameters or characteristics.
- For instance, the synchrotron processes is mediated by ambient magnetic fields, the inverse Compton (IC) process is mediated by the local radiation field, and bremsstrahlung is controlled by the local density of target nuclei.
- Whether, in a particular energy range, a population of cosmic ray electrons belonging to a distinct astrophysical source, emits more in synchrotron or IC or bremsstrahlung is therefore controlled by the relative sizes of the object's magnetic field and light field energy densities and its gas density.
- In addition, different processes tend to produce radiation in different energy regimes.

General problem

- There can be other, non-radiative processes shaping the spectrum of the non-thermal particle populations including: the shape of the injection distribution of particles, non-radiative energy loss processes like ionisation, and transport processes which can mean that particles escape a particular region before they lose much energy to radiation.
- Altogether, we can hope to model the combined effects of the many different radiative and non-radiative energy loss processes and transport processes to reconstruct the in situ particle distribution of an astrophysical source.

Evolution of particle distributions with energy loss and/or transport

- The competition between acceleration, in situ energy loss, and escape leads to the formation of the broadband energy distribution – i.e., spectrum – of non-thermal particles inside astro-physical objects.
- For definiteness, we will focus on describing the evolution of CR electron (and/or positron) distributions, though much of the formalism is general enough to carry across to describing the evolution of hadronic CR populations.
- Let us write the in situ differential spectrum of electrons as

$$\frac{dN_e}{dE} = f_e(E)$$

Evolution of particle distributions with energy loss and/or transport

- We specify that there is some local acceleration process that injects particles of energy E at a rate $Q_e(E)$.
- Particles cool due to radiative and non-radiative losses at an energy-dependent rate $\dot{E}(E)$.
- Under these losses, the number of particles around some injection energy E_{inj}, f(E_{inj}), will tend to decrease but, particles are conserved, so there will be a corresponding increase of the particle number density around the energy $E_{\text{inj}} - \int \dot{E} dt$.
- In general, we can construct a continuity equation in the phase space of the energy parameter which is just a line: $$\frac{\partial f_e}{\partial t}(E,t) = Q_e(E,t) - \frac{\partial}{\partial E}\left(\dot{E}(E) f_e(E,t)\right)$$

Continuity equation

$$\frac{\partial f_e}{\partial t}(E,t) = Q_e(E,t) - \frac{\partial}{\partial E}\left(\dot{E}(E) f_e(E,t)\right)$$

- Note that here we have labelled Q_e a source term but, in general, such terms could also be negative ‘sinks’.
- An important example of such a sink is in the case of particle escape from the region under consideration as parameterised by some (in general) energy-dependent escape time $\tau_{esc}(E)$:

$$Q_{\rm esc}(E) = -\frac{f_e(E)}{\tau_{\rm esc}(E)}$$

Continuity equation

- We generalise the continuity equation to account for such escape as: $$\frac{\partial f_e}{\partial t}(E,t) = Q_e(E,t) - \frac{\partial}{\partial E}\left(\dot{E}(E) f_e(E,t)\right) - \frac{f_e(E)}{\tau_{\rm esc}(E)}$$
- In analogy with the escape time, we can also define an energy-dependent energy loss timescale:

$$\tau_{\rm loss}(E) = \frac{E}{|\dot{E}(E)|}$$

Continuity equation

- There are some limiting cases of Equation 6 from which we can garner some very useful physical insights.
- First, in the situation that we are in steady state – which requires that $t \gg \tau_{esc}, \tau_{loss}$ we shall have that: $$\frac{\partial f_e}{\partial t}(E,t) = 0$$
- Then, in steady state, two useful limits are defined by the 'thick target' and 'escape-dominated' regimes, which correspond to the two limits $\tau_{loss}/\tau_{esc} \ll 1$ and $\tau_{loss}/\tau_{esc} \gg 1$, respectively.

Continuity equation

- In the steady state, thick target limit, we get
$$\frac{\partial}{\partial E}\left(\dot{E}(E)f_e(E,t)\right) = Q_e(E)$$
with solution:
$$f_e(E)_{\rm thick} = \frac{1}{\dot{E}(E)}\int_E^{\infty} Q_e(E')dE'.$$

- This case is also called 'calorimetric' because all of the power injected into freshly accelerated particles (as represented by the $Q_e(E)$ is dissipated in in situ losses (typically radiative losses).

Continuity equation

- The above has a particularly simple solution in the case that the injection spectrum is (tolerably approximated as) monoenergetic $Q_e(E) \propto \delta(E - E_0)$ so the integral evaluates to a unit step function Θ:

$$f_e(E)_{\mathrm{thick,mono}} = \frac{1}{\dot{E}(E)}\Theta(E_0 - E).$$

- Another useful limit of the thick target case is for the case that both the injection spectrum and energy loss rate can be characterised as power laws in particle energy:

$$Q_e(E) \propto E^{\gamma}_{\mathrm{inj}} \quad \text{and} \quad \dot{E}(E) \propto E^{\gamma}_{\mathrm{cool}}$$

- From this:

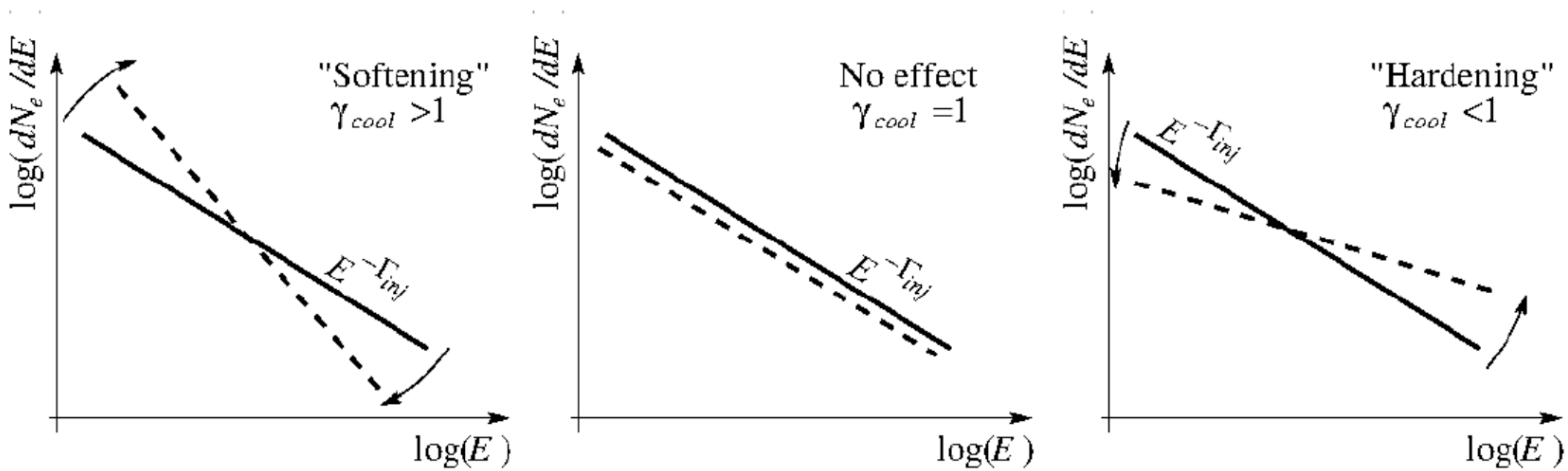
$$\begin{aligned} f_{\mathrm{thick}}(E) &\propto E^{-\gamma_{\mathrm{cool}}} \int_E^{\infty} E'^{\gamma_{\mathrm{inj}}} dE' \\ &\propto E^{\gamma_{\mathrm{inj}}-\gamma_{\mathrm{cool}}+1} \qquad ; \qquad (\gamma_{\mathrm{inj}} < -1) \end{aligned}$$

Continuity equation

- $$\begin{aligned} f_{\text{thick}}(E) &\propto E^{-\gamma_{\text{cool}}} \int_E^{\infty} E'^{\gamma_{\text{inj}}} dE' \\ &\propto E^{\gamma_{\text{inj}} - \gamma_{\text{cool}} + 1} \qquad ; \qquad (\gamma_{\text{inj}} < -1) \end{aligned}$$ leads to three cases:
  - $\gamma_{cool} > 1$: here particle losses lead to 'pile up' at low energy, i.e., the steady state spectrum is steepened or 'softened' with respect to the injection spectrum
  - $\gamma_{cool} = 1$: here particle losses are independent of energy so such losses do not change the shape of the spectrum
  - $\gamma_{cool} < 1$: here particle losses become more severe at low energies, preferentially 'washing out' the lower energy part of the distribution with the implication that the steady state spectrum is flatter or 'harder' than the injection distribution. An example of a process with this sort of behaviour is ionisation.

Continuity equation

Credit: Neronov, *High-Energy Astrophysics*



Examples of how a cooled steady state particle distribution can be hardened, softened, or left unchanged with respect to the injection distribution according to the energy dependence of the cooling or energy loss process

Continuity equation

- In the alternative, escape-dominated steady state limit of the continuity equation we have $f_e(E)_{\rm esc} \simeq \tau_{\rm esc}(E) Q_e(E).$
- Such a simple treatment of escape of particles as characterised by a single (energy-dependent) escape time is also known as the 'leaky box'.
- In general, for any resolved region of finite extent there will be a position-dependence to the escape time that requires a more sophisticated treatment.

Continuity equation

- This equations grants us a qualitative understanding of the spectrum of CRs in the Galactic plane and detected at Earth.
- We saw in lecture I that the spectrum of CR ions is, to a reasonable approximation, given by a power law $f_{\mathrm{CR,MW}} \propto E^{-2.7}$
- At the same time, there is a well-founded theoretical expectation that CR sources like supernova remnants accelerate CRs into power-law distributions: $Q_{\mathrm{CR}} \propto E^{-(2.0+\delta)}$ where here $\delta \sim 0.1 - 0.4$.
- Substituting $f_{\mathrm{CR,MW}}$ and Q_{CR} into above see that…
-

Continuity equation

$$
\begin{aligned}
\tau_{\rm esc,MW}(E)_{\rm esc} &\simeq \frac{f_{\rm CR,MW}}{Q_e(E)} \\
&\sim E^{-2.7+(2+\delta)} \\
&= E^{\delta-0.7}
\end{aligned}
$$

- and thus : $\tau_{\rm esc,MW}(E) \propto (E^{-0.3} - E^{-0.6}).$
- Therefore the escape or 'confinement' time of CR ions in the Galaxy declines with increasing energy.
- In other words, higher energy CRs tend to escape from the Galactic disc more quickly than lower energy ones

Emission from a broadband particle distribution

- CR populations tend to cover wide swathes of energy.
- Such 'broadband' distributions generically lead to photon emission over very wide energy ranges, particularly when there are multiple radiative loss processes at play.
- For instance, consider CR electrons around an energy scale of $E_e \sim$ GeV in a typical patch of the Milky Way's interstellar medium (ISM).
- On the ~ few μG magnetic field, these particles will emit synchrotron radiation with a characteristic energy scale

$$E_{\gamma,\mathrm{synch}} \simeq 4 \times 10^{-8}\ \mathrm{eV} \left(\frac{E_e}{\mathrm{GeV}}\right)^2 \left(\frac{\mathrm{B}}{\mu\mathrm{G}}\right)$$

Emission from a broadband particle distribution

- The same electrons will inverse Compton upscatter ambient light from a typical photon energy $E_{\gamma,0}$ up to an energy:

$$E_{\gamma,\mathrm{IC}} \simeq \gamma_e^2 \; E_{\gamma,0} = \left(\frac{E_e}{m_e}\right)^2 \; E_{\gamma,0} \simeq 4\ \mathrm{MeV}\ \left(\frac{E_e}{\mathrm{GeV}}\right)^2 \left(\frac{E_{\gamma,0}}{\mathrm{eV}}\right)$$

- …and emit (relativistic) bremsstrahlung γ-rays via collisions with ambient gas at characteristic energy: $E_{\gamma,\mathrm{brems}} \sim \frac{1}{2} E_e \sim \ \mathrm{GeV}$

- Thus, even just considering CR electrons near GeV, we can expect radiated photons in a range ~ $10^{-7} \rightarrow 10^{9}$ eV, i.e., 16 orders of magnitude in energy!

Emission from a broadband particle distribution

- Accounting for the fact that the emitting electrons themselves can be expected to cover a range of energy, we see that the band of emitted photons will cover an even wider range of energies.

Distribution of emitted photons

- Return, for the moment, to the simple case of a monoenergetic electron distribution at energy E_e and say that this leads to the emission of photons (via some generic radiative process) with a spectrum given by $\Phi(E_e, E_\gamma)$.
- From $\Phi(E_e, E_\gamma)$. we can determine the spectrum of photons emitted by a distribution of electrons dNe(Ee)/dEe as
$$\frac{dN_\gamma}{dE_\gamma}(E_\gamma) = \int \frac{dN_e}{dE_e}(E_e)\Phi(E_e, E_\gamma)dE_e$$
- As we already saw analogously for the lecture on hadronic gamma-ray emission, the photon spectrum arising from a monoenergetic electron population is identical to the probability density for an electron of E_e to radiate a photon of E_γ:
$$\frac{dP}{dE_\gamma}(E_e \to E_\gamma) \equiv \Phi(E_e, E_\gamma)$$

Distribution of emitted photons

- Now, let us assume that the characteristic energy of photons emitted in our generic radiative process $E_{\gamma,*}$ scales as some power, σ, of the energy of the radiating electron: $E_{\gamma,*} \propto E_e^{\sigma}$
- This might seem like a fairly particular assumption but it, again, turns out to be fairly generic to radiative processes of relevance for high-energy astrophysics. So, for instance, we have that $\sigma \rightarrow 2$ for synchrotron and IC emission and $\sigma \rightarrow 1$ for bremsstrahlung emission.

- Approximate the photon spectrum emitted by a monoenergetic electron spectrum as $$\Phi(E_e, E_\gamma) = c_3\delta(E_\gamma - c_2 E_e^\sigma)$$ where c_3 and c_2 are to be determined.
- We can normalise this equation by requiring that, integrating over the entire photon spectrum at fixed E_e we recover the total power lost by electrons: $$\int_0^{E_e} \Phi(E_e, E_\gamma) E_\gamma = -\frac{dE_e}{dt}(E_e) \equiv c_1 E_e^{\gamma_{\rm cool}}$$
- From which we derive that c_1, c_2 and c_3 are related as: $$c_3 = \frac{c_1}{c_2} E_e^{\gamma_{\rm cool}-\sigma}$$

Distribution of emitted photons

- We therefore have: $$\frac{dN_\gamma}{dE_\gamma} \propto \int_{E_\gamma}^{\infty} \frac{c_1}{c_2} E_e^{\gamma_{\rm cool}-\sigma} \delta(E_\gamma - c_2 E_e^\sigma) dE_e$$
- …where we have assumed that the electron spectrum is a power law.
- After a change of variables ($u = c_2 E_e^\sigma$), , this gives us a general relationship between the spectral index, γ_e of the emitting electron population (or, in general, the spectral index of the non-thermal particle population) and the photon spectral index (which we here call α): $$\frac{dN_\gamma}{dE_\gamma} \propto E_\gamma^{\frac{1+\gamma_{\rm cool}-2\sigma+\gamma_e}{\sigma}} \equiv E_\gamma^\alpha$$

Distribution of emitted photons

radiative mechanism	$\gamma_{\rm cool}$	σ	photon index (α)
synchrotron	2	2	$\frac{\gamma_e - 1}{2}$
IC	2	2	$\frac{\gamma_e - 1}{2}$
bremsstrahlung	1	1	γ_e
π^0 (hadronic)	1	1	$\sim \gamma_p$

Spectral breaks

- Above we discussed the limiting cases that, in steady state, either particle escape dominantly shapes the in situ spectrum or energy loss dominantly shapes the spectrum.
- In general, escape time and energy loss time can be functions of energy and, indeed, the loss (or cooling) times associated to different radiative (or non-radiative) loss processes can be different.
- This means that, scanning over the energy covered by the spectrum of non-thermal particles, different processes can be dominant for different energy ranges.
- This translates into different spectral indices governing the in situ spectrum in each of these ranges, with ‘spectral breaks’ smoothly connecting between the different regions; note that these breaks are present even in the case that the injection distribution is described by a pure, featureless power law. These ‘spectral breaks’ in the non-thermal particle distributions, moreover, are reflected in spectral breaks in the emitted radiation.

Spectral breaks

- Consider a concrete case: we have in situ acceleration of a power law distribution of electrons in a certain astrophysical source.
- There is a steady flow of gas out of this source – a 'wind' – that, along with the thermal gas, advects away the ambient particles over some characteristic timescale. The action of the wind, therefore, implies an energy-independent escape time τ_{esc} = const.
- At the same time, imagine that there is an ambient magnetic field which leads to electron energy loss via synchrotron radiation. The characteristic timescale of these synchrotron losses scales like $\tau_{\mathrm{loss,synch}} \propto 1/E_e$.

Spectral breaks

- Given these different scalings, note that there will be a critical electron energy above which $\tau_{\mathrm{loss,synch}} < \tau_{\mathrm{esc}}$ (so the spectrum will be loss-dominated or in the thick-target limit) and below which $\tau_{\mathrm{loss,synch}} > \tau_{\mathrm{esc}}$ (so the spectrum will be escape-dominated.)
- In the low-energy, escape-dominated regime we will have that the steady state spectrum is identical in spectral index to the injection distribution: for $Q_e(E) \propto E^{\gamma_{\mathrm{inj}}}$ and $\tau_{\mathrm{esc}} = \mathrm{const}$, we will have $f_e(E) = Q_e \tau_{\mathrm{esc}} \propto E^{\gamma_{\mathrm{inj}}}$.

Spectral breaks

- On the other hand, in the high-energy, loss-dominated regime we will have that the steady state spectrum is steepened by 1 in spectral index with respect to the injection distribution: for $Q_e(E) \propto E^{\gamma_{\rm inj}}$ we will have: $f_e(E) \propto E^{\gamma_{\rm inj}-1}$.
- Between the two regimes, the electron distribution steepens by 1: $\Delta\gamma_e = \gamma_{\rm e,high} - \gamma_{\rm e,low} = (\gamma_{\rm inj} - 1) - \gamma_{\rm inj} = -1.$
- This break will be reflected in the spectrum of the emitted synchrotron radiation.
- Note the important point here that the distribution of electrons does synchrotron emission over its entire energy range even though synchrotron itself is only determining the shape of the in situ electron spectrum above the break.
- Thus synchrotron plays two, distinct roles: its shape the electron spectrum due to the energy losses it implies (but it is only the dominant 'shaper' above the break!) and it generates radiation that we can detect (both above and below the break)

Spectral breaks

Using the results presented in the table, the change of the emitted synchrotron radiation across the break is:

$\Delta\alpha = (\gamma_{\mathrm{e,high}} - 1)/2 - (\gamma_{\mathrm{e,low}} - 1)/2 = ((\gamma_{\mathrm{inj}} - 1) - 1)/2 - (\gamma_{\mathrm{inj}} - 1)/2 = -1/2$

Spectral breaks

- Finally, we deal with some nomenclature that is commonly used in high-energy astrophysics:
  - dN_γ/dE_γ is called the 'differential spectrum'
  - $E_\gamma\, dN_\gamma/dE_\gamma$ is called the 'photon spectrum',
  - $E_\gamma^2\, dN_\gamma/dE_\gamma$ is called the 'spectral energy distribution'.